Objectives:

- Use the Extreme Value Theorem to find a function's absolute extrema on a closed interval.
- Use the Second Derivative Test to categorize critical points.
- Practice finding absolute and local extrema.

From yesterday's project:

The Extreme Value Theorem

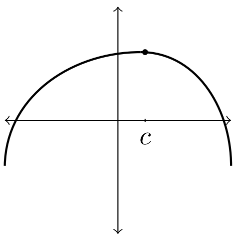
Examples: Let's see how we can find absolute maximum and minimum values, if they exist, for the following functions over the given intervals.

1. $f(x) = x - \ln(x)$ on $[0, 2]$

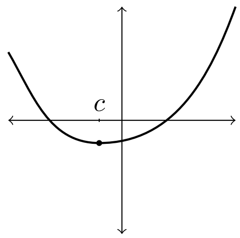
2. $g(t) = t^3 - 3t^2 - 20$ on $[3, 6]$

3. $h(x) = 3 - |x - 1|$ on $[0, 5]$

More on Local Extrema:
What about functions not over a closed interval? Then we can't compare function values of critical points to function values at endpoints. How can we figure out if a critical point is a maximum, a minimum, or neither?
So far we have one tool to classify critical points, the first derivative test.
Now, we introduce another option:



$f(c)$ is
$f'(c)$
$f''(c)$



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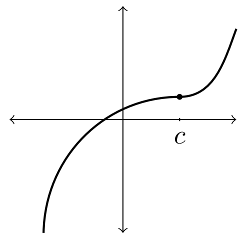

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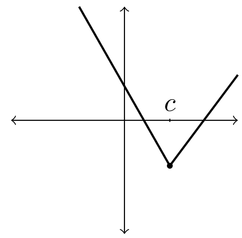

$f(c)$ is a
$f'(c)$
$f''(c)$

The Second Derivative Test:
If f is continuous near c, then:

(a) If $f'(c) = 0$ and $f''(c) > 0$

(b) If $f'(c) = 0$ and $f''(c) < 0$

(c) If $f'(c) = 0$ and $f''(c) = 0$

(d) If $f'(c)$ or $f''(c)$ is undefined

Examples: Find and classify all critical points of the following functions:

1. $f(x) = x^4 - 4x^3$

2. $p(x) = x + \sqrt{1-x}$

3. $g(t) = t^4$

Finding Inflection Points:

An inflection point is a point on a curve where ______________________________ . We can also think of an inflection point as ____ .

To find an inflection point:

1. Find where the second derivative is ____________________ .
2. Find the sign of the _______________ on each interval between the points from step 1.
3. If ____, then $\big(c, f(c)\big)$ is an inflection point.

Example: Find and classify all critical points of $f(x) = 3xe^{-2x}$, as well as finding its inflection points.

More Examples!

1. Consider $s(r) = 2\pi r^2 + \frac{80}{r}$ on the domain $(0, \infty)$. Find, if possible, the local and absolute extrema and inflection points of $s(r)$.

2. Given the following graph of $f'(x)$, find the local extrema, absolute extrema, and inflection points of $f(x)$. [The domain of f is $(0, 4)$.] Use this information to graph $f(x)$.